

Two-axis spin squeezing of two-component BEC via a continuous driving

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In two-component BEC, the one-axis twisting Hamiltonian leads to spin squeezing with the limitation that scales with the number of atoms as $N^{-\frac{2}{3}}$. We propose a scheme to transform the one-axis twisting Hamiltonian into a two-axis twisting Hamiltonian, resulting in enhanced spin squeezing $\propto N^{-1}$ approaching the Heisenberg limit. Instead of pulse sequences, only one continuous driving field is required to realizing such transforming, thus the scheme is promising for experiment realizations, to an one-axis twisting Hamiltonian. Quantum information processing and quantum metrology may benefit from this method in the future.

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Introduction. Squeezed spin states (SSSs) [1–3], whose concept was firstly established by Kitagawa and Ueda [1], are entangled quantum states of an ensemble of spin systems. The SSS attracted considerable attention due to their significant roles in studying many-particle entanglement [4–9] and applications for high-precision measurements [2, 10–16]. In the original proposal [1], there are two distinguished ways to produce SSS, one interaction in the form as χJ_x^2 is known as one-axis twisting (OAT), the other one in the form as $\chi(J_+^2 - J_-^2)$ is known as two-axis twisting (TAT). The OAT scheme just can reduce the noise limit to the scale as $N^{-\frac{2}{3}}$ where N is atom number, while the TAT can produce the SSS with the squeezing parameter scaling with N^{-1} [1]. Both in theory and experiment, most schemes can only produce effective OAT-type spin-spin interactions, such as direct atom collisions in Bose-Einstein condensates (BEC) [17–19], indirect spin-spin interaction by quantum nondemolition measurement [20–25] and cavity feedback [26, 27].

The two-component BEC is a very promising system for OAT SSSs [28–30], and it has been demonstrated in experiments recently [17, 18, 31, 32]. It holds two main advantages, including the considerable long coherence time and the strong atom-atom interaction, is very potential for future applications. Therefore, various efforts are dedicated to realizing the TAT type Hamiltonian to enhance the squeezing in such system [33–36]. One of the proposals [34] transforms an OAT Hamiltonian into an effective TAT Hamiltonian by applying a large number of repeated Rabi pulses, which would be sensitive to the accumulation of control errors. In another scheme [35], one or two global rotation pulses are applied at an appropriate evolution time and with optimized rotation angles, which reduces the number of pulses greatly, but requires a long evolution time to achieve the optimal squeezing and the control pulse is spin number dependent.

In this paper, we propose a scheme to transform the OAT into the effective TAT spin squeezing in BEC by continuous coherent driving. Under the driving, the spin state is rotating along the direction perpendicular to the twisting axis, then generate the effect Hamiltonian as mixed OAT and TAT. By carefully choosing and tuning the amplitude and frequency of the driving field, pure TAT can be realized and a Heisenberg limited noise reduction $\propto N^{-1}$ is obtained. Compared with the previous scheme [34], our proposal uses a continuous field instead of pulse sequences, which is more friendly for experiments. What's more, our scheme is spin number independent and needs a shorter evolution time compared with [35]. The principle of continuous driving transformed the OAT to the TAT can also be applied to other systems, such as the cavity feedback [26, 27] and spin state dependent geometry phase [37] induced OAT.

Theoretical Model. According to Refs. [17, 18], the two-component BEC with a coherent driving can be described by the following Hamiltonian

$$H = \chi J_x^2 + \Omega(t) J_z. \quad (1)$$

Here $J_\mu = \sum_{k=1}^N \sigma_\mu^k / 2$ in terms of the Pauli matrices σ_μ^k ($\mu = x, y, z$) is the collective angular momentum operator for the spin ensemble consisting of N atoms. The first term of the Hamiltonian is the OAT induced by atom-atom collisions, with χ the nonlinear interaction strength. The second term is the external classical laser driving with magnetic field along the z -axis. For the continuous driving, we assume $\Omega(t) = g \cos(\omega t)$, where g and ω are the strength and frequency of the driving field, respectively.

Transform the Hamiltonian (1) into the interaction representation, we get

$$\begin{aligned} H_I &= e^{i \int_0^t \Omega(\tau) J_z d\tau} \chi J_x^2 e^{-i \int_0^t \Omega(\tau) J_z d\tau} \\ &= \frac{\chi}{4} (e^{2igC} J_+^2 + e^{-2igC} J_-^2 + J_+ J_- + J_- J_+), \quad (2) \end{aligned}$$

where $C = \int_0^t \cos(\omega t) dt = \frac{\sin(\omega t)}{\omega}$ and $J_\pm = J_x \pm iJ_y$. According to the Jacobi-Anger expansion $e^{iz \sin \theta} =$

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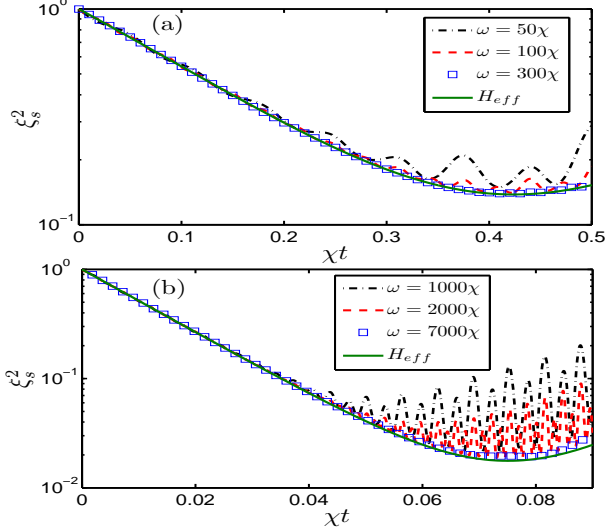


FIG. 1: (Color online) (a) Spin squeezing parameter as a function of the evolution time for different frequencies of the driving field with the number of atoms $N = 10$. The frequencies are $\omega = 50\chi$ (dot-dashed dark line), $\omega = 100\chi$ (dashed red line), $\omega = 300\chi$ (blue squares) and H_{eff} (solid blue line). (b) Same as (a) except for $N = 100$ with the frequencies corresponding to $\omega = 1000\chi$ (dot-dashed dark line), $\omega = 2000\chi$ (dashed red line), $\omega = 7000\chi$ (blue squares) and H_{eff} (solid blue line). $\frac{g}{\omega} = 0.906$ in both (a) and (b).

$\sum_{n=-\infty}^{\infty} \mathcal{J}_n(z) e^{in\theta}$ where $\mathcal{J}_n(z)$ is the n -th Bessel function of the first kind, the terms in Eq. (2) can be expanded as

$$e^{\pm 2igC} = e^{\pm i\frac{2g}{\omega} \sin(\omega t)} = \sum_{n=-\infty}^{\infty} \mathcal{J}_n(\pm \frac{2g}{\omega}) e^{in\omega t}. \quad (3)$$

When ω is quite large ($\omega \gg N\chi$), the high-order terms with $n \neq 0$ are neglected due to the rotating wave approximation. Then, the Hamiltonian becomes

$$H_I' \simeq \frac{\chi}{2} [(A+1)J_x^2 - (A-1)J_y^2], \quad (4)$$

where the constant $A = \mathcal{J}_0(\frac{2g}{\omega})$. Therefore, the external driving field leads to the twisting effect along both x and y directions. This can be interpreted intuitively as the rotation of spins perpendicular to the axis of OAT (x -axis) diverted the twisting axis.

Rewriting the Hamiltonian by adding a constant $\frac{\chi}{2}(A-1)J^2$ (which is conserved during the dynamics), we obtain a mixture of an OAT Hamiltonian and a TAT Hamiltonian as

$$H_I'' = \frac{\chi}{2} (3A-1)J_x^2 + \frac{\chi}{2} (1-A)(J_x^2 - J_z^2). \quad (5)$$

Tune the values of g and ω to be $\frac{g}{\omega} = 0.906$, then $A = \mathcal{J}_0(\frac{2g}{\omega}) = \frac{1}{3}$ and the effective Hamiltonian of the system becomes

$$H_{eff} = \frac{\chi}{3} (J_x^2 - J_z^2). \quad (6)$$

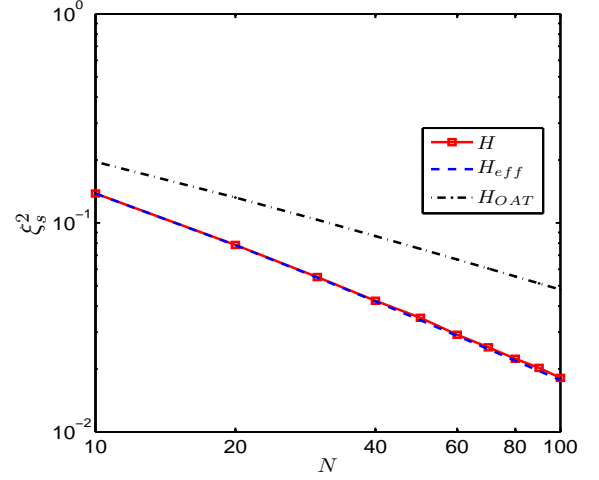


FIG. 2: (Color online) The optimal spin squeezing plotted against the number of atoms N for dynamics generated by H (square red line), H_{eff} (dashed blue line) and H_{OAT} (dotted-dashed line).

Obviously, H_{eff} exhibits the well-known TAT Hamiltonian. Alternatively, we can also adjust the parameters to satisfy $\mathcal{J}_0(\frac{2g}{\omega}) = -\frac{1}{3}$, then we obtain another TAT Hamiltonian

$$H'_{eff} = \frac{\chi}{3} (J_y^2 - J_z^2). \quad (7)$$

Therefore, the OAT Hamiltonian can be transformed into the TAT Hamiltonian by tuning the amplitude and frequency of the driving field. Similar ideas have been studied by Law et al. [38], where the underlying physics is the same with the continuous driving method studied here. In that work, a steady field are applied for coherent controlling of the SSS, which is consist with our model with $\omega = 0$, effectively generate a mixture of OAT and TAT. It's worth noting that the effective nonlinear interaction strength reduces to $1/3$, which is due to the cancellation of part of spin squeezing when rotating of the squeezing direction.

Numerical results. To verify our idea above, we study the spin squeezing numerically by solving the evolution of spin state. The initial state is chosen to be a coherent spin state (CSS) [1] along the y axis, which is $|\varphi(0)\rangle = 2^{-J} \sum_{k=0}^{2J} i^k \sqrt{(2J)!/(k)!(2J-k)!} |J, J-k\rangle$ satisfying $J_y|\varphi(0)\rangle = J|\varphi(0)\rangle$ with $J = N/2$, where $|J, k\rangle$ are the eigenstates of J_z . We choose squeezing parameter $\xi_s^2 \equiv 4\min(\Delta J_{\vec{n}_\perp})^2/N$ [1] to quantify the squeezing, where \vec{n}_\perp refers to the direction perpendicular to the mean spin direction and the minimization is taken over all such directions.

In Fig. 1, we plot the spin squeezing parameter as a function of the evolution time for different driving frequency ω but fixed the ratio that $\frac{g}{\omega} = 0.906$ to obtain optimized TAT. The results for both the $N = 10$ (a) and $N = 100$ (b) are agrees well with the effective TAT Hamiltonian [Eq. 6]. There are fast oscillations of the ξ_s^2

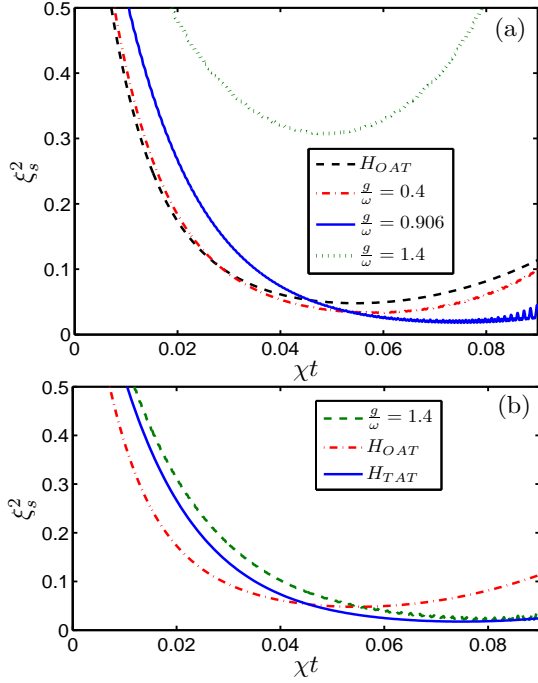


FIG. 3: (Color online) (a) Spin squeezing parameter as a function of the evolution time for different $\frac{g}{\omega}$ with $N = 100$. The curves are $\frac{g}{\omega} = 0.4$ (dotted-dashed red line), $\frac{g}{\omega} = 0.906$ (solid blue line), $\frac{g}{\omega} = 1.4$ (dotted green line) and H_{OAT} (dashed dark line). The initial state is along the y axis. (b) Spin squeezing parameter as a function of the evolution time with $\frac{g}{\omega} = 1.4$ (dashed green line), H_{OAT} (dot-dashed red line) and H_{TAT} (solid blue line) with $N = 100$. The initial state is along the x axis.

for small ω , which is attributed to the high-order terms in the Jacobi-Anger expansion [Eq. 3] when $\omega/\chi \gg N$ is not satisfied. For example, the oscillation period for $\omega = 50\chi$ is about $T = 0.06/\chi$, corresponding to $\omega T \approx \pi$ which consist with the period of high order terms. Therefore, higher frequency ω is favorable for larger number of atoms. In addition, we find that when the number of atoms N increases, it needs a shorter time to reach the optimal squeezing, and the time is already much shorter than the scheme [35].

Next, we investigate how the optimal squeezing of H without approximation scales with N . We plot the optimal spin squeezing (minimum value of ξ_s^2) as a function of the number of atoms in Fig. 2. The red solid line corresponding to H shows the optimal spin squeezing parameter $\xi_s^2 \propto N^{-1}$ which is the well-known Heisenberg limited noise reduction, and it agrees well with H_{eff} (the blue dashed line). For comparison, we also present the $N^{-\frac{2}{3}}$ scaling of the OAT Hamiltonian $H_{OAT} = \chi J_x^2$.

Although the optimal TAT should satisfy $\frac{g}{\omega} = 0.906$, we could expect the enhanced spin squeezing by continuous driving field is robust against imperfection parameters, since the external driving field could lead to mixture of OAT and TAT effectively [Eq. 5]. In Fig. 3(a),

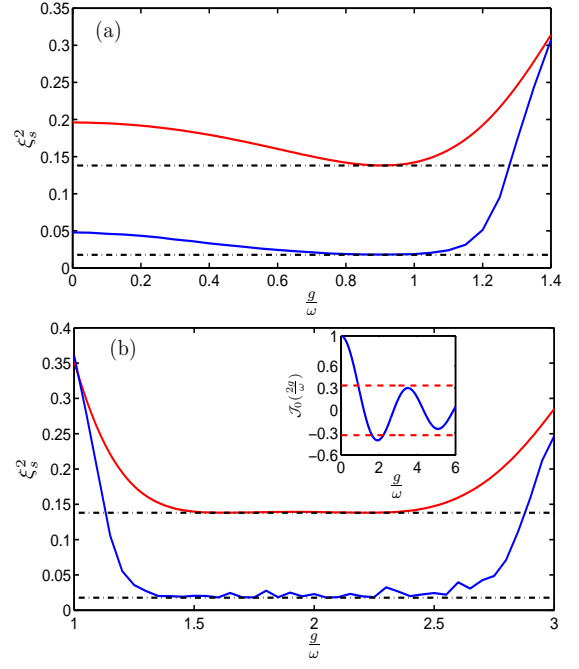


FIG. 4: (Color online) (a) The optimal spin squeezing as a function of $\frac{g}{\omega}$ with $N = 10$ (solid red line), $N = 100$ (solid blue line). The two horizontal lines correspond to the optimal squeezing of the TAT Hamiltonian $\xi_s^2 = 0.1381$ ($N = 10$) and $\xi_s^2 = 0.0177$ ($N = 100$). The initial state is along the y axis. (b) The same as (a) except for the initial state along the x axis. The inset shows the Bessel function $J_0(2g/\omega)$ varying as g/ω , and the horizontal lines indicate $J_0(2g/\omega) = 1/3, -1/3$ respectively.

we show the squeezing parameter as a function of the evolution time for different $\frac{g}{\omega}$ with $N = 100$, the dynamics under H_{OAT} is also presented for comparison. We can find that at $\frac{g}{\omega} = 0.4$ the optimal squeezing generated by H is 0.02805. It is better than that generated by H_{OAT} (0.0479) while worse than that generated by H_{TAT} (0.0177). But at $\frac{g}{\omega} = 1.4$, the squeezing is even worse than H_{OAT} , which is owing to $J_0(2.8)$ approaching $-\frac{1}{3}$, then the Hamiltonian H is close to $H'_{eff} = \frac{\chi}{3}(J_y^2 - J_z^2)$. If we change the initial state correspondingly along x -axis, which is $|\varphi(0)\rangle = 2^{-J} \sum_{k=0}^{2J} \sqrt{\frac{(2J)!}{(k)!(2J-k)!}} |J, J-k\rangle$ satisfying $J_x|\varphi(0)\rangle = J|\varphi(0)\rangle$, the effect of this Hamiltonian approaches the idea TAT, as shown in Fig. 3(b). Therefore, our scheme can always enhance the OAT Hamiltonian to achieve better SSS even though the achievable $\frac{g}{\omega}$ is deviated from optimal value.

Finally, we plot the optimal spin squeezing parameter of the Hamiltonian H as a function of $\frac{g}{\omega}$ in Fig. 4(a) with the initial state being a CSS along the y axis and in Fig. 4(b) with the initial state being a CSS along the x axis. In Fig. 4(a), the minimum value equals to the optimal squeezing of the TAT Hamiltonian appears at $\frac{g}{\omega} \simeq 0.906$, which agrees with the optimal condition.

The rapid growth of ξ_s^2 for $\frac{\Omega}{\omega} > 1.2$ is due to the Hamiltonian changing to H'_{eff} . In Fig. 4(b), it shows a section of gentle variance which almost equals to the optimal squeezing of the TAT Hamiltonian. There are two points $\frac{\Omega}{\omega} \simeq 1.626, 2.221$ which make $\mathcal{J}_0(\frac{2\Omega}{\omega}) = -\frac{1}{3}$, and the minimum value of the Bessel function $\mathcal{J}_0(z)$ between the two points is about -0.4027 [inset of Fig. 4(b)] which has no large variance comparing with $-1/3$. Thus, there is a quite large range approaching the TAT squeezing, which is favorable for experiments.

Conclusions. We have proposed a scheme to transform an OAT Hamiltonian into a TAT type by applying a continuous driving field. We find that a TAT Hamiltonian can be obtained by tuning the ratio of the driving field

amplitude to the frequency, and even though at other more achievable values of $\frac{\Omega}{\omega}$, the squeezing performance of our scheme is more better than the OAT scheme. Compared with the previous proposals [34][35], our scheme is more friendly for experiments and faster. Since the continuous driving field can be manipulated relatively easily, we believe it is realizable with current techniques as reported in Ref. [17, 18].

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